

# On the equivalence of the Langevin and auxiliary field quantization methods for absorbing dielectrics.

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Recently two methods have been developed for the quantization of the electromagnetic field in general dispersing and absorbing linear dielectrics. The first is based upon the introduction of a quantum Langevin current in Maxwell's equations [T. Gruner and D.-G. Welsch, *Phys. Rev. A* **53**, 1818 (1996); Ho Trung Dung, L. Knöll, and D.-G. Welsch, *Phys. Rev. A* **57**, 3931 (1998); S. Scheel, L. Knöll, and D.-G. Welsch, *Phys. Rev. A* **58**, 700 (1998)], whereas the second makes use of a set of auxiliary fields, followed by a canonical quantization procedure [A. Tip, *Phys. Rev. A* **57**, 4818 (1998)]. We show that both approaches are equivalent.

## I. INTRODUCTION

With the advent of modern optical materials, such as optical fibers and photonic crystals, the problem of quantization of the electromagnetic field in dielectrics has become an important subject and much activity has been taking place in this field. Quantization is required to describe the decay of embedded atoms (for specific cases, see [1,2]), the Casimir effect [3,4] and other nonclassical phenomena such as the propagation of entangled states through dielectrics [5]. We also mention the generation of X-ray transition radiation by fast electrons traveling through layered dielectrics [6].

In the case of linear conservative dielectrics, quantization is well-known for systems, where the permittivity (electric permeability)  $\varepsilon$  is a real constant [7] or a real function of space,  $\varepsilon = \varepsilon(\mathbf{r})$  [8,9]. Nonlinear dielectrics are discussed in [10]. For dispersing and truly absorbing media the situation is more complicated, because the permittivity is a complex function of frequency and varies with space in general, i.e.,  $\varepsilon = \varepsilon(\mathbf{r}, \omega)$ . Progress in this field has been fairly recent.

Two basic approaches can be distinguished. The first is based on the Hopfield model of a bulk dielectric [11]. The quantized electromagnetic field is coupled to a material system described by a harmonic oscillator model, and the

Hamiltonian of the total system is diagonalized [12]. A drawback is that it becomes rather cumbersome if spatial inhomogeneities are present [13]. Also the identification of the permittivity is not trivial [14,15].

The second approach starts off from the classical phenomenological Maxwell equations, featuring a general spatially inhomogeneous, complex, frequency-dependent permittivity  $\varepsilon(\mathbf{r}, \omega)$  satisfying the Kramers-Kronig relations. It has the advantage that the really measured values of the permittivity can be used for the theoretical description of quantized light in media. For example, in the case of photonic crystals made up from dielectric objects (scatterers) in a conservative, homogeneous background (such as vacuum),  $\varepsilon(\mathbf{r}, \omega)$  is known at the outset. Absorption is often undesirable, in particular if one is interested in band-gap phenomena. In fact absorption prohibits the formation of the latter [16]. Band-gap photonic crystals offer many interesting technological applications [17] but require a large dielectric contrast between the scatterers and background. This can be accomplished by using small metal spheres showing a Drude-type behavior, where the real part of the permittivity can acquire large negative values [18]. However, such systems are always somewhat absorbing. On the positive side, absorption may be advantageous in the case of transition radiation, where it can be used to suppress undesired frequencies [6].

There have been two concepts of quantization of the phenomenological Maxwell field for general dispersing and absorbing linear dielectrics. The first (referred to as LN concept) is based upon the introduction of Langevin noise current (and charge) densities, as dictated by the fluctuation-dissipation theorem, into the classical Maxwell equations, which can then be transferred to quantum theory by conversion of the electromagnetic field quantities into operators. After some earlier work [14,19], restricted to specific simple geometries, a general formalism was put forward by some of us [20–22]. In this scheme the dyadic Green's function associated with the classical (inhomogeneous) Helmholtz equation plays a prominent role. Its properties come into play by deriving the equal-time commutators for the fields, given those of the noise current operator. In Ref. [21] the case of a planar interface and in Ref. [2,23] the spontaneous de-

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cay in a spherical cavity is worked out but more involved situations can also be handled. Basically the Green's function of the classical problem must be calculated. For this, general methods and a variety of specific examples are considered in Ref. [24]. Efficient methods have been developed (such as an adaptation of the KKR approach of solid state physics) for the photonic crystal case [25].

The second concept (referred to as AF concept) developed by one of us (AT) [26,27] also starts off from the classical phenomenological Maxwell equations. Here, the introduction of a set of auxiliary fields (instead of a noise current) allows the replacement of Maxwell's equations, which feature a time convolution term relating the polarization to the electric field, by a new set of equations for the combined set of electromagnetic and auxiliary fields but without time convolutions. For the so extended system, a conserved quantity, bilinear in all fields, generalizing the electromagnetic energy, exists. Maxwell's equations are retrieved by setting the initial auxiliary fields equal to zero. The system can then be quantized and the conserved quantity becomes the Hamiltonian. But now the initial auxiliary fields, being operators, can no longer be set equal to zero.

Although the two concepts of quantization of the phenomenological Maxwell field look quite different at first glance, they are fully equivalent, as we show in the present paper. For this purpose, we first review in Section II the basic formulas of the LN concept. After reviewing the AF concept, we then prove in Section III that the AF concept precisely leads to the noise current operator in the LN concept. Some concluding remarks are given in Section IV.

## II. THE LANGEVIN NOISE METHOD

Starting point is the set of the classical macroscopic Maxwell equations for the electromagnetic field in an absorbing linear dielectric without free charges and currents

$$\partial_t \mathbf{D}(\mathbf{r}, t) = \partial_{\mathbf{r}} \times \mathbf{H}(\mathbf{r}, t), \quad (2.1)$$

$$\partial_t \mathbf{B}(\mathbf{r}, t) = -\partial_{\mathbf{r}} \times \mathbf{E}(\mathbf{r}, t), \quad (2.2)$$

$$\partial_{\mathbf{r}} \cdot \mathbf{D}(\mathbf{r}, t_0) = 0, \quad (2.3)$$

$$\partial_{\mathbf{r}} \cdot \mathbf{B}(\mathbf{r}, t_0) = 0, \quad (2.4)$$

$$\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}(\mathbf{r}, t), \quad (2.5)$$

$$\mathbf{P}(\mathbf{r}, t) = \varepsilon_0 \int_{t_0}^t ds \chi(\mathbf{r}, t-s) \mathbf{E}(\mathbf{r}, s) + \mathbf{P}_n(\mathbf{r}, t), \quad (2.6)$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t), \quad (2.7)$$

where the initial time  $t_0$  may be set to  $t_0 = -\infty$ . Introducing the Fourier transform of the electric-field strength according to

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \int_{-\infty}^{+\infty} d\omega \exp[-i\omega t] \mathbf{E}(\mathbf{r}, \omega) \\ &= \int_0^{+\infty} d\omega \exp[-i\omega t] \mathbf{E}(\mathbf{r}, \omega) + \text{c.c.}, \end{aligned} \quad (2.8)$$

and the Fourier transforms of the other fields accordingly, Eqs. (2.1) – (2.7) lead to

$$\partial_{\mathbf{r}} \times \mathbf{H}(\mathbf{r}, \omega) = -i\omega \mathbf{D}(\mathbf{r}, \omega), \quad (2.9)$$

$$\partial_{\mathbf{r}} \times \mathbf{E}(\mathbf{r}, \omega) = i\omega \mathbf{B}(\mathbf{r}, \omega), \quad (2.10)$$

$$\partial_{\mathbf{r}} \cdot \mathbf{D}(\mathbf{r}, \omega) = 0, \quad (2.11)$$

$$\partial_{\mathbf{r}} \cdot \mathbf{B}(\mathbf{r}, \omega) = 0, \quad (2.12)$$

$$\mathbf{D}(\mathbf{r}, \omega) = \varepsilon_0 \varepsilon(\mathbf{r}, \omega) \mathbf{E}(\mathbf{r}, \omega) + \mathbf{P}_n(\mathbf{r}, \omega), \quad (2.13)$$

$$\mathbf{B}(\mathbf{r}, \omega) = \mu_0 \mathbf{H}(\mathbf{r}, \omega), \quad (2.14)$$

where

$$\varepsilon(\mathbf{r}, \omega) = 1 + \chi(\mathbf{r}, \omega), \quad (2.15)$$

$$\chi(\mathbf{r}, \omega) = \int_0^{\infty} dt \exp[i\omega t] \chi(\mathbf{r}, t). \quad (2.16)$$

Note that for absorbing media

$$\varepsilon(\mathbf{r}, \omega) = \varepsilon_R(\mathbf{r}, \omega) + i\varepsilon_I(\mathbf{r}, \omega), \quad \varepsilon_I(\mathbf{r}, \omega) \geq 0. \quad (2.17)$$

In the LN concept [20–22,28], Eqs. (2.9) – (2.14) [or Eqs. (2.1) – (2.7)] are considered as a set of equations for the electromagnetic field supplemented with a noise polarization  $\mathbf{P}_n(\mathbf{r}, \omega)$  [29]. Its introduction arises from the necessity to fulfill the fluctuation-dissipation theorem, because macroscopic electrodynamics is a statistical theory. In a classical theory the noise term can only be dropped in the zero-temperature limit,  $T \rightarrow 0$ , whereas in quantum theory it is always present due to vacuum noise. From these arguments, the operator-valued fields (indicated with hats) in quantum electrodynamics can be regarded as obeying Eqs. (2.9) – (2.14),

$$\partial_{\mathbf{r}} \times \hat{\mathbf{B}}(\mathbf{r}, \omega) = -i\frac{\omega}{c^2} \varepsilon(\mathbf{r}, \omega) \hat{\mathbf{E}}(\mathbf{r}, \omega) + \mu_0 \hat{\mathbf{j}}_n(\mathbf{r}, \omega), \quad (2.18)$$

$$\partial_{\mathbf{r}} \times \hat{\mathbf{E}}(\mathbf{r}, \omega) = i\omega \hat{\mathbf{B}}(\mathbf{r}, \omega), \quad (2.19)$$

$$\partial_{\mathbf{r}} \cdot \varepsilon_0 \varepsilon(\mathbf{r}, \omega) \hat{\mathbf{E}}(\mathbf{r}, \omega) = \hat{\rho}_n(\mathbf{r}, \omega), \quad (2.20)$$

$$\partial_{\mathbf{r}} \cdot \hat{\mathbf{B}}(\mathbf{r}, \omega) = 0, \quad (2.21)$$

$$\hat{\mathbf{D}}(\mathbf{r}, \omega) = \varepsilon_0 \varepsilon(\mathbf{r}, \omega) \hat{\mathbf{E}}(\mathbf{r}, \omega) + \hat{\mathbf{P}}_n(\mathbf{r}, \omega), \quad (2.22)$$

where  $\hat{\rho}_n(\mathbf{r}, \omega)$  and  $\hat{\mathbf{j}}_n(\mathbf{r}, \omega)$  are the noise charge and current densities,

$$\hat{\rho}_n(\mathbf{r}, \omega) = -\partial_{\mathbf{r}} \cdot \hat{\mathbf{P}}_n(\mathbf{r}, \omega), \quad (2.23)$$

$$\hat{\mathbf{j}}_n(\mathbf{r}, \omega) = -i\omega \hat{\mathbf{P}}_n(\mathbf{r}, \omega). \quad (2.24)$$

Quantization is accomplished by relating the current to bosonic vector fields according to ( $\varepsilon_0 \mu_0 = c^{-2}$ )

$$\begin{aligned} \hat{\mathbf{j}}_n(\mathbf{r}, \omega) &= \frac{\omega}{\mu_0 c^2} \sqrt{\frac{\hbar}{\pi \varepsilon_0}} \varepsilon_I(\mathbf{r}, \omega) \hat{\mathbf{f}}(\mathbf{r}, \omega) \\ &= \omega \sqrt{\frac{\hbar \varepsilon_0}{\pi}} \varepsilon_I(\mathbf{r}, \omega) \hat{\mathbf{f}}(\mathbf{r}, \omega), \end{aligned} \quad (2.25)$$

$$[\hat{\mathbf{f}}(\mathbf{r}, \omega), \hat{\mathbf{f}}^\dagger(\mathbf{r}', \omega')] = \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega') \mathbf{U}, \quad (2.26)$$

$$[\hat{\mathbf{f}}(\mathbf{r}, \omega), \hat{\mathbf{f}}(\mathbf{r}', \omega')] = [\hat{\mathbf{f}}^\dagger(\mathbf{r}, \omega), \hat{\mathbf{f}}^\dagger(\mathbf{r}', \omega')] = 0, \quad (2.27)$$

where  $\mathbf{U}$  is the unit  $3 \times 3$  matrix. The fields  $\hat{\mathbf{f}}(\mathbf{r}, \omega)$  represent the fundamental variables of the overall system. In the Heisenberg picture they evolve as

$$\hat{\mathbf{f}}(\mathbf{r}, \omega, t) = \exp[-i\omega(t - t')] \hat{\mathbf{f}}(\mathbf{r}, \omega, t'), \quad (2.28)$$

which is governed by the Hamiltonian

$$\hat{H} = \int d\mathbf{r} \int_0^\infty d\omega \hbar \omega \hat{\mathbf{f}}^\dagger(\mathbf{r}, \omega) \hat{\mathbf{f}}(\mathbf{r}, \omega). \quad (2.29)$$

The commutation relations (2.26) and (2.27) imply that

$$\begin{aligned} & [\hat{\mathbf{j}}_n(\mathbf{r}, \omega), \hat{\mathbf{j}}_n^\dagger(\mathbf{r}', \omega')] \\ &= \left( \frac{\omega}{\mu_0 c^2} \right)^2 \frac{\hbar}{\pi \varepsilon_0} \varepsilon_I(\mathbf{r}, \omega) \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega') \mathbf{U} \\ &= \frac{\hbar \omega^2}{\pi} \varepsilon_0 \varepsilon_I(\mathbf{r}, \omega) \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega') \mathbf{U}. \end{aligned} \quad (2.30)$$

From Eqs. (2.18) and (2.19) it follows that  $\hat{\mathbf{E}}(\mathbf{r}, \omega)$  satisfies the equation

$$\begin{aligned} & \partial_{\mathbf{r}} \times \partial_{\mathbf{r}} \times \hat{\mathbf{E}}(\mathbf{r}, \omega) - \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}, \omega) \hat{\mathbf{E}}(\mathbf{r}, \omega) \\ &= [\mathbf{H}_0 - \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}, \omega) \mathbf{U}] \cdot \hat{\mathbf{E}}(\mathbf{r}, \omega) = i\omega \mu_0 \hat{\mathbf{j}}_n(\mathbf{r}, \omega) \end{aligned} \quad (2.31)$$

( $\mathbf{H}_0 = \partial_{\mathbf{r}} \times \partial_{\mathbf{r}} \times = \partial_{\mathbf{r}} \partial_{\mathbf{r}} - \partial_{\mathbf{r}}^2 \mathbf{U}$ ). Inversion of Eq. (2.31) and the use of Eq. (2.19) yields

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = i\omega \mu_0 \int d\mathbf{s} \mathbf{G}(\mathbf{r}, \mathbf{s}, \omega) \cdot \hat{\mathbf{j}}_n(\mathbf{s}, \omega), \quad (2.32)$$

$$\begin{aligned} \hat{\mathbf{B}}(\mathbf{r}, \omega) &= (i\omega)^{-1} \partial_{\mathbf{r}} \times \hat{\mathbf{E}}(\mathbf{r}, \omega) \\ &= \mu_0 \partial_{\mathbf{r}} \times \int d\mathbf{s} \mathbf{G}(\mathbf{r}, \mathbf{s}, \omega) \cdot \hat{\mathbf{j}}_n(\mathbf{s}, \omega). \end{aligned} \quad (2.33)$$

Here,  $\mathbf{G}$  is the classical Green function (actually a second-rank symmetric tensor) that satisfying the equation

$$\left\{ \partial_{\mathbf{r}} \partial_{\mathbf{r}} - \left[ \partial_{\mathbf{r}}^2 + \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}, \omega) \right] \mathbf{U} \right\} \cdot \mathbf{G}(\mathbf{r}, \mathbf{s}, \omega) = \delta(\mathbf{r} - \mathbf{s}) \mathbf{U}. \quad (2.34)$$

Note that  $\mathbf{G}$  corresponds to the operator  $[\partial_{\mathbf{r}} \partial_{\mathbf{r}} - \{\partial_{\mathbf{r}}^2 + \omega^2 \varepsilon(\mathbf{r}, \omega)/c^2\} \mathbf{U}]^{-1}$ , which exists as a bounded Hilbert-space operator if

$$\lim_{|\mathbf{r}| \rightarrow \infty} [\varepsilon(\mathbf{r}, \omega) - 1] = i0_+, \quad (2.35)$$

automatically fixing the boundary conditions of  $\mathbf{G}$  at infinity (cf. Ref. [16]).

The electric-field strength operator in the Schrödinger picture can then be represented in the form of [cf. Eq. (2.8)]

$$\hat{\mathbf{E}}(\mathbf{r}) = \int_0^\infty d\omega \hat{\mathbf{E}}(\mathbf{r}, \omega) + \text{H.c.}, \quad (2.36)$$

and the other field operators accordingly. Using the properties of  $\mathbf{G}$  it can be verified that the standard equal-time commutation relations of quantum electrodynamics fields are fulfilled [20–22, 28]. Since the latter do not depend on  $\varepsilon(\mathbf{r}, \omega)$ , the case that  $\varepsilon_I(\mathbf{r}, \omega)$  (approximately) vanishes in a certain  $\omega$ -interval can be handled by means of a limiting procedure. It is worth noting that the LN method has the advantage that arbitrary inhomogeneous, anisotropic, amplifying or magnetic matter can easily be included in the formalism [28].

### III. THE AUXILIARY FIELD METHOD AND ITS RELATION TO THE LANGEVIN NOISE METHOD

#### A. Classical formalism

The AF method [27] starts from the zero-temperature classical Maxwell equations ( $\mathbf{P}_n = 0$ ) and complements them with appropriately chosen auxiliary fields. In order to facilitate a comparison with the LN method, we shall use a setup where only Fourier components for positive argument are used and in addition we shall use a different gauge for the fields. We assume that  $\chi(\mathbf{r}, t = 0) = 0$ , which can be verified from linear response theory. It excludes instantaneous surges at the initial time. Then, with  $\chi'(\mathbf{r}, t) = \partial_t \chi(\mathbf{r}, t)$ ,

$$\begin{aligned} \partial_t \mathbf{E}(\mathbf{r}, t) &= c^2 \partial_{\mathbf{r}} \times \mathbf{B}(\mathbf{r}, t) - \int_{-\infty}^t ds \chi'(\mathbf{r}, t - s) \mathbf{E}(\mathbf{r}, s) \\ &= c^2 \partial_{\mathbf{r}} \times \mathbf{B}(\mathbf{r}, t) - \mathbf{J}(\mathbf{r}, t), \end{aligned} \quad (3.1)$$

where  $\mathbf{J}(\mathbf{r}, t) = \partial_t \mathbf{P}(\mathbf{r}, t)$  is the polarization current density. Since  $\chi(\mathbf{r}, 0) = 0$  we have [the factor 2 arises from changing the range of the  $\lambda$ -integral from  $\mathbb{R}$  in Ref. [27] to  $[0, \infty]$ ]

$$\chi(\mathbf{r}, t) = 2 \int_0^\infty d\lambda \lambda^{-1} \sin(\lambda t) \nu(\mathbf{r}, \lambda), \quad (3.2)$$

$$\chi'(\mathbf{r}, t) = 2 \int_0^\infty d\lambda \cos(\lambda t) \nu(\mathbf{r}, \lambda), \quad (3.3)$$

where  $\nu(\mathbf{r}, \lambda) \geq 0$  for absorbing systems considered here. Note that

$$\varepsilon_I(\mathbf{r}, \lambda) = \frac{\pi}{\lambda} \nu(\mathbf{r}, \lambda) \quad (3.4)$$

( $\lambda \geq 0$ ). Next we define

$$\mathbf{F}_1(\mathbf{r}, t) = \sqrt{\varepsilon_0} \mathbf{E}(\mathbf{r}, t), \quad (3.5)$$

$$\mathbf{F}_3(\mathbf{r}, t) = \frac{1}{\sqrt{\mu_0}} \mathbf{B}(\mathbf{r}, t) \quad (3.6)$$

and introduce the auxiliary fields

$$\begin{aligned} \mathbf{F}_2(\mathbf{r}, \lambda, t) &= -\sqrt{\varepsilon_0} \sigma(\mathbf{r}, \lambda) \\ &\times \int_{-\infty}^t ds \sin \lambda(t - s) \mathbf{E}(\mathbf{r}, s), \end{aligned} \quad (3.7)$$

$$\begin{aligned} \mathbf{F}_4(\mathbf{r}, \lambda, t) &= -\sqrt{\varepsilon_0} \sigma(\mathbf{r}, \lambda) \\ &\times \int_{-\infty}^t ds \cos \lambda(t-s) \mathbf{E}(\mathbf{r}, s), \end{aligned} \quad (3.8)$$

where

$$2\nu(\mathbf{r}, \lambda) = \sigma(\mathbf{r}, \lambda)^2, \quad \sigma(\mathbf{r}, \lambda) \geq 0, \quad (3.9)$$

and note that

$$\mathbf{F}_2(\mathbf{r}, \lambda, -\infty) = \mathbf{F}_4(\mathbf{r}, \lambda, -\infty) = 0. \quad (3.10)$$

It can be proved [27] that the set of equations

$$\begin{aligned} \partial_t \mathbf{F}_1(\mathbf{r}, t) &= c \partial_{\mathbf{r}} \times \mathbf{F}_3(\mathbf{r}, t) \\ &+ \int_0^\infty d\lambda \sigma(\mathbf{r}, \lambda) \mathbf{F}_4(\mathbf{r}, \lambda, t), \end{aligned} \quad (3.11)$$

$$\partial_t \mathbf{F}_2(\mathbf{r}, \lambda, t) = \lambda \mathbf{F}_4(\mathbf{r}, \lambda, t), \quad (3.12)$$

$$\partial_t \mathbf{F}_3(\mathbf{r}, t) = -c \partial_{\mathbf{r}} \times \mathbf{F}_1(\mathbf{r}, t), \quad (3.13)$$

$$\partial_t \mathbf{F}_4(\mathbf{r}, \lambda, t) = -\lambda \mathbf{F}_2(\mathbf{r}, \lambda, t) - \sigma(\mathbf{r}, \lambda) \mathbf{F}_1(\mathbf{r}, t) \quad (3.14)$$

together with the initial conditions (3.10) is equivalent to Maxwell's equations, and the quantity

$$\begin{aligned} \mathcal{E} &= \frac{1}{2} \int d\mathbf{r} [\mathbf{E}(\mathbf{r}, t)^2 + \mathbf{B}(\mathbf{r}, t)^2] \\ &+ \frac{1}{2} \int d\mathbf{r} \int_0^\infty d\lambda [\mathbf{F}_2(\mathbf{r}, \lambda, t)^2 + \mathbf{F}_4(\mathbf{r}, \lambda, t)^2] \end{aligned} \quad (3.15)$$

is conserved in time. Note that  $\mathcal{E}$  coincides with the electromagnetic energy for vanishing  $\chi$ .

Our aim is to find a quantized version of Eqs. (3.11) – (3.14). Since the initial condition (3.10) then loses its meaning, we now drop it. Setting

$$\mathbf{F}_0(\mathbf{r}, \lambda, t) = \mathbf{F}_4(\mathbf{r}, \lambda, t) - i\mathbf{F}_2(\mathbf{r}, \lambda, t), \quad (3.16)$$

we have

$$\partial_t \mathbf{F}_0(\mathbf{r}, \lambda, t) = -i\lambda \mathbf{F}_0(\mathbf{r}, \lambda, t) - \sigma(\mathbf{r}, \lambda) \mathbf{F}_1(\mathbf{r}, t). \quad (3.17)$$

Its solution can be written as

$$\begin{aligned} \exp(i\lambda t) \mathbf{F}_0(\mathbf{r}, \lambda, t) \\ = \mathbf{F}'_0(\mathbf{r}, \lambda) - \sigma(\mathbf{r}, \lambda) \int_{-\infty}^t ds \exp(i\lambda s) \mathbf{F}_1(\mathbf{r}, s). \end{aligned} \quad (3.18)$$

Since the second term on the right hand side vanishes as  $t \rightarrow -\infty$ , we obtain (for the limit, see the Appendix)

$$\mathbf{F}'_0(\mathbf{r}, \lambda) = \lim_{t \rightarrow -\infty} \exp(i\lambda t) \mathbf{F}_0(\mathbf{r}, \lambda, t). \quad (3.19)$$

On the other hand, setting  $t=0$  in Eq. (3.18) yields

$$\begin{aligned} \mathbf{F}'_0(\mathbf{r}, \lambda) &= \mathbf{F}_0(\mathbf{r}, \lambda, 0) \\ &+ \sigma(\mathbf{r}, \lambda) \int_{-\infty}^0 ds \exp(i\lambda s) \mathbf{F}_1(\mathbf{r}, s). \end{aligned} \quad (3.20)$$

With

$$\mathbf{F}'_0(\mathbf{r}, \lambda, t) = \exp(-i\lambda t) \mathbf{F}'_0(\mathbf{r}, \lambda) \quad (3.21)$$

Eqs. (3.11) – (3.14) are then replaced by

$$\partial_t \mathbf{D}(\mathbf{r}, t) = \partial_{\mathbf{r}} \times \mathbf{H}(\mathbf{r}, t), \quad (3.22)$$

$$\partial_t \mathbf{B}(\mathbf{r}, t) = -\partial_{\mathbf{r}} \times \mathbf{E}(\mathbf{r}, t), \quad (3.23)$$

$$\partial_t \mathbf{F}'_2(\mathbf{r}, \lambda, t) = \lambda \mathbf{F}'_4(\mathbf{r}, \lambda, t), \quad (3.24)$$

$$\partial_t \mathbf{F}'_4(\mathbf{r}, \lambda, t) = -\lambda \mathbf{F}'_2(\mathbf{r}, \lambda, t), \quad (3.25)$$

where

$$\begin{aligned} \mathbf{D}(\mathbf{r}, t) &= \varepsilon_0 \mathbf{E}(\mathbf{r}, t) \\ &+ \varepsilon_0 \int_{-\infty}^t ds \chi(\mathbf{r}, t-s) \mathbf{E}(\mathbf{r}, s) + \mathbf{P}'(\mathbf{r}, t). \end{aligned} \quad (3.26)$$

Here,

$$\begin{aligned} \mathbf{P}'(\mathbf{r}, t) &= \sqrt{\varepsilon_0} \int d\lambda \lambda^{-1} \sigma(\mathbf{r}, \lambda) \\ &\times [\cos(\lambda t) \mathbf{F}'_2(\mathbf{r}, \lambda) + \sin(\lambda t) \mathbf{F}'_4(\mathbf{r}, \lambda)] \end{aligned} \quad (3.27)$$

can be regarded as being the noise polarization, with

$$\begin{aligned} \mathbf{J}'(\mathbf{r}, t) &= \partial_t \mathbf{P}'(\mathbf{r}, t) = \sqrt{\varepsilon_0} \int d\lambda \sigma(\lambda) \\ &\times [\sin(\lambda t) \mathbf{F}'_2(\mathbf{r}, \lambda) - \cos(\lambda t) \mathbf{F}'_4(\mathbf{r}, \lambda)]. \end{aligned} \quad (3.28)$$

being the associated noise current density. In Eqs. (3.27) and Eqs. (3.28) we have set  $\mathbf{F}'_0 = \mathbf{F}'_4 - i\mathbf{F}'_2$  with  $\mathbf{F}'_2$  and  $\mathbf{F}'_4$  real. Note that the equations of motion for the primed auxiliary fields are decoupled from those of the electromagnetic fields.

## B. Hamilton formalism and quantization

We can interpret Eqs. (3.22) – (3.25) as a set of equations suitable for transferring to quantum theory. The equations of motion for the  $\mathbf{F}'$ -fields describe harmonic motions and are readily quantized, thus leading to a quantum noise contribution in the field equations. However, there actually exists a Hamiltonian formalism, generating the full set of field equations, which can then be quantized [26,27]. The basic equations are Eqs. (3.11) – (3.14), which can be written in the compact form of

$$\partial_t \mathbf{F} = \mathbf{N} \mathbf{F} = \begin{pmatrix} 0 & \mathbf{N}_{em} \\ \mathbf{N}_{me} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{F}_e \\ \mathbf{F}_m \end{pmatrix}, \quad (3.29)$$

where  $\mathbf{F}_e$  consists of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  and  $\mathbf{F}_m$  of  $\mathbf{F}_3$  and  $\mathbf{F}_4$ .

In order to show the equivalence to the LN method, we adopt a generalization of the temporal or Weyl gauge (the T-gauge in Ref. [26]) instead of the generalized Coulomb gauge in Ref [27]. Thus we set

$$\mathbf{F}_e = -\partial_t \boldsymbol{\xi}_e = -\partial_t \begin{pmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2 \end{pmatrix}, \quad (3.30)$$

$$\mathbf{F}_m = \begin{pmatrix} \mathbf{F}_3 \\ \mathbf{F}_4 \end{pmatrix} = -\mathbf{N}_{me} \begin{pmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2 \end{pmatrix}, \quad (3.31)$$

which implies that

$$\mathbf{E} = -\partial_t \varepsilon_0^{-1/2} \boldsymbol{\xi}_1, \quad (3.32)$$

$$\mathbf{B} = \partial_{\mathbf{r}} \times \varepsilon_0^{-1/2} \boldsymbol{\xi}_1, \quad (3.33)$$

$$\mathbf{F}_2 = -\partial_t \boldsymbol{\xi}_2, \quad (3.34)$$

$$\mathbf{F}_4 = \sigma \boldsymbol{\xi}_1 + \lambda \boldsymbol{\xi}_2. \quad (3.35)$$

Obviously,  $\mathbf{A} = \varepsilon_0^{-1/2} \boldsymbol{\xi}_1$  is the vector potential. The Lagrangian generating the equations of motion is

$$\begin{aligned} L &= \frac{1}{2} \langle \partial_t \boldsymbol{\xi}_e, \partial_t \boldsymbol{\xi}_e \rangle - \frac{1}{2} \langle \mathbf{N}_{me} \boldsymbol{\xi}_e, \mathbf{N}_{me} \boldsymbol{\xi}_e \rangle \\ &= \frac{1}{2} \langle \partial_t \boldsymbol{\xi}_e, \partial_t \boldsymbol{\xi}_e \rangle - \frac{1}{2} \langle \mathbf{H}_e \boldsymbol{\xi}_e, \boldsymbol{\xi}_e \rangle, \end{aligned} \quad (3.36)$$

where the (real) inner product is defined according to

$$\begin{aligned} \langle \mathbf{f}, \mathbf{g} \rangle &= \varepsilon_0 \int d\mathbf{r} [\mathbf{f}_1(\mathbf{r}) \cdot \mathbf{g}_1(\mathbf{r}) \\ &\quad + \int_0^\infty d\lambda \mathbf{f}_2(\mathbf{r}, \lambda) \cdot \mathbf{g}_2(\mathbf{r}, \lambda)]. \end{aligned} \quad (3.37)$$

The canonical momentum is

$$\boldsymbol{\pi}_e = \begin{pmatrix} \boldsymbol{\pi}_1(\mathbf{r}) \\ \boldsymbol{\pi}_2(\mathbf{r}, \lambda) \end{pmatrix} = \begin{pmatrix} \partial_t \boldsymbol{\xi}_1 \\ \partial_t m \boldsymbol{\xi}_2 \end{pmatrix}, \quad (3.38)$$

thus  $\mathbf{E} = -\varepsilon_0^{-1/2} \boldsymbol{\pi}_1$ , and the Hamiltonian becomes

$$\begin{aligned} H &= \frac{1}{2} \langle \boldsymbol{\pi}_e, \boldsymbol{\pi}_e \rangle + \frac{1}{2} \langle \mathbf{N}_{me} \boldsymbol{\xi}_e, \mathbf{N}_{me} \boldsymbol{\xi}_e \rangle \\ &= \frac{1}{2} \langle \boldsymbol{\pi}_e, \boldsymbol{\pi}_e \rangle + \frac{1}{2} \langle \mathbf{H}_e \boldsymbol{\xi}_e, \boldsymbol{\xi}_e \rangle. \end{aligned} \quad (3.39)$$

Here (cf. Ref. [27])

$$\mathbf{H}_e = \mathbf{N}_{me}^* \mathbf{N}_{me} = \begin{pmatrix} c^2 \mathbf{H}_0 + \chi'(\mathbf{r}, 0) & \int_0^\infty d\lambda \lambda \sigma(\mathbf{r}, \lambda) \cdots \\ \lambda \sigma(\mathbf{r}, \lambda) & \lambda^2 \end{pmatrix} \quad (3.40)$$

where  $\mathbf{H}_0 = \partial_{\mathbf{r}} \partial_{\mathbf{r}} - \partial_{\mathbf{r}}^2 \mathbf{U}$  is a non-negative selfadjoint operator in  $\mathcal{H}_e = \mathcal{H}_1 \oplus \mathcal{H}_2$ ,  $\mathcal{H}_1 = L^2(\mathbb{R}^3, d\mathbf{r}; \mathbb{R}^3)$ ,  $\mathcal{H}_2 = L^2(\mathbb{R}^3 \times \mathbb{R}^+, d\mathbf{r} d\lambda; \mathbb{R}^3)$ . Note that the off-diagonal terms in Eq. (3.40) describe the coupling of the electromagnetic and auxiliary fields, but that there is also a shift term  $\chi'(\mathbf{r}, 0)$ , and it can be verified that  $H$  equals the conserved quantity  $\mathcal{E}$ .

Quantization is achieved by setting

$$[\hat{\boldsymbol{\xi}}_1(\mathbf{r}), \hat{\boldsymbol{\pi}}_1(\mathbf{r}')] = i\hbar \delta(\mathbf{r} - \mathbf{r}') \mathbf{U}, \quad (3.41)$$

$$[\hat{\boldsymbol{\xi}}_2(\mathbf{r}, \lambda), \hat{\boldsymbol{\pi}}_2(\mathbf{r}', \lambda')] = i\hbar \delta(\mathbf{r} - \mathbf{r}') \delta(\lambda - \lambda') \mathbf{U}, \quad (3.42)$$

all other commutators being zero. The Heisenberg equations of motion are given according to Eqs. (3.11) – (3.14) [or Eq. (3.29)]. Defining  $\hat{\mathbf{F}}_0(\mathbf{r}, \lambda, t)$  according to Eq. (3.16), it again satisfies the (operator-valued) Eq. (3.17) with the solution according to Eq. (3.18):

$$\begin{aligned} \hat{\mathbf{F}}_0(\mathbf{r}, \lambda, t) &= \exp[-i\lambda t] \hat{\mathbf{F}}_0'(\mathbf{r}, \lambda) \\ &\quad - \sigma(\mathbf{r}, \lambda) \int_{-\infty}^t ds \exp[-i\lambda(t-s)] \hat{\mathbf{F}}_1(\mathbf{r}, s), \end{aligned} \quad (3.43)$$

where, according to Eq. (3.19) together with Eq. (3.16),

$$\hat{\mathbf{F}}_0'(\mathbf{r}, \lambda) = \hat{\mathbf{F}}_4'(\mathbf{r}, \lambda) - i\hat{\mathbf{F}}_2'(\mathbf{r}, \lambda), \quad (3.44)$$

with  $\hat{\mathbf{F}}_{2,4}'(\mathbf{r}, \lambda, t)$  and  $\hat{\mathbf{F}}_{2,4}'(\mathbf{r}, \lambda)$  selfadjoint. Insertion of Eq. (3.43) in the equations of motion results into the operator-valued Eqs. (3.22) – (3.27). According to Eq. (3.28), the operator of the noise current density reads

$$\begin{aligned} \hat{\mathbf{J}}'(\mathbf{r}, t) &= \sqrt{\varepsilon_0} \int d\lambda \sigma(\lambda) \\ &\quad \times [\sin(\lambda t) \hat{\mathbf{F}}_2'(\mathbf{r}, \lambda) - \cos(\lambda t) \hat{\mathbf{F}}_4'(\mathbf{r}, \lambda)]. \end{aligned} \quad (3.45)$$

Using Eqs. (3.41) and (3.42), we find the (equal-time) commutation relations

$$[\hat{\mathbf{F}}_4'(\mathbf{r}, \lambda), \hat{\mathbf{F}}_2'(\mathbf{r}', \lambda')] = -i\hbar \lambda \delta(\mathbf{r} - \mathbf{r}') \delta(\lambda - \lambda') \mathbf{U} \quad (3.46)$$

and

$$\begin{aligned} &[\hat{\mathbf{J}}'(\mathbf{r}, \lambda), \hat{\mathbf{J}}'^{\dagger}(\mathbf{r}', \lambda')] \\ &= \frac{\hbar \lambda^2}{\pi} \varepsilon_0 \varepsilon_I(\mathbf{r}, \lambda) \delta(\mathbf{r} - \mathbf{r}') \delta(\lambda - \lambda') \mathbf{U}. \end{aligned} \quad (3.47)$$

Thus, we identify  $\hat{\mathbf{J}}'(\mathbf{r}, \lambda)$  with the Langevin noise current in Section II,

$$\hat{\mathbf{J}}'(\mathbf{r}, \lambda) = \hat{\mathbf{j}}_n(\mathbf{r}, \lambda). \quad (3.48)$$

With this choice, Eq. (3.47) exactly equals Eq. (2.30). Hence, the AF formalism is equivalent with the LN formalism. Introducing creation and annihilation operators according to

$$\hat{\mathbf{F}}_2'(\mathbf{r}, \lambda) = i(\hbar \lambda / 2)^{1/2} [\hat{\mathbf{b}}(\mathbf{r}, \lambda) - \hat{\mathbf{b}}^{\dagger}(\mathbf{r}, \lambda)], \quad (3.49)$$

$$\hat{\mathbf{F}}_4'(\mathbf{r}, \lambda) = (\hbar \lambda / 2)^{1/2} [\hat{\mathbf{b}}(\mathbf{r}, \lambda) + \hat{\mathbf{b}}^{\dagger}(\mathbf{r}, \lambda)], \quad (3.50)$$

we have

$$[\hat{\mathbf{b}}(\mathbf{r}, \lambda), \hat{\mathbf{b}}^{\dagger}(\mathbf{r}', \lambda')] = \delta(\mathbf{r} - \mathbf{r}') \delta(\lambda - \lambda') \mathbf{U}, \quad (3.51)$$

and hence

$$\hat{\mathbf{f}}(\mathbf{r}, \lambda) = -\hat{\mathbf{b}}(\mathbf{r}, \lambda) = -(2\hbar \lambda)^{-1/2} \hat{\mathbf{F}}_0'(\mathbf{r}, \lambda). \quad (3.52)$$

## IV. DISCUSSION

In the LN formalism, the basic ingredient is the identification of the (usually discarded) noise polarization in Eq. (2.6) or Eq. (2.13) as the fundamental field variable of the theory from which all properties of the electromagnetic field can be derived by means of Eqs. (2.23), (2.24), (2.32), (2.33), and Maxwell's equations (2.9) – (2.14). For any temperature, the fluctuation-dissipation theorem is then satisfied, and the classical, statistical noise polarization can be regarded as being an operator-valued

quantity in quantum theory. Then one can show that the correct (equal-time) QED commutation relations are satisfied.

The AF formalism starts from Maxwell's equations without the noise polarization. Instead, auxiliary fields are introduced whose equations of motion eventually decouple from those of the electromagnetic fields leaving behind a source term in Ampère's law. The formalism can then be cast into a Hamiltonian form, which, upon quantization, features a noise current with the same commutation properties as in the LN formalism. For ease of comparison, a generalized temporal gauge has been adopted, but the actual choice does of course not affect the equal-time commutator  $[\hat{\mathbf{E}}(\mathbf{r}), \hat{\mathbf{B}}(\mathbf{r}')].$

It should be pointed out that Eq. (2.31), which plays a crucial role in the LN concept, can be related to the eigenvalue problem associated with  $H_e$  in the AF formalism. Indeed, from Eqs. (3.11) – (3.14) we have

$$\partial_t^2 \mathbf{F}_e(t) = -H_e \mathbf{F}_e(t). \quad (4.1)$$

Thus, in the stationary solution  $\mathbf{F}_e(t) = \exp[-i\omega t] \mathbf{F}_e(\omega)$ ,  $\mathbf{F}_e(\omega)$  solves the eigenvalue problem

$$H_e \mathbf{F}_e(\omega) = \omega^2 \mathbf{F}_e(\omega), \quad (4.2)$$

where the first of these equations,

$$[c^2 H_0 + \chi'(0)] \mathbf{F}_1(\omega) + \int_0^\infty d\lambda \lambda \sigma(\lambda) \mathbf{F}_2(\lambda, \omega) = \omega^2 \mathbf{F}_1(\omega), \quad (4.3)$$

corresponds to Eq. (2.31), given the relation (3.19) between  $\mathbf{F}_0$  and  $\mathbf{F}'_0$ . As shown in the Appendix, this relation has a precise scattering theoretical background.

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## APPENDIX: THE RELATION BETWEEN $\mathbf{F}_0$ AND $\mathbf{F}'_0$

The existence of the limit

$$\mathbf{F}'_0(\mathbf{r}, \lambda) = \lim_{t \rightarrow -\infty} \exp(i\lambda t) \mathbf{F}_0(\mathbf{r}, \lambda, t) \quad (A1)$$

suggests that  $\mathbf{F}_0(\mathbf{r}, \lambda, t)$  is asymptotically free, i.e., its motion becomes decoupled from that of the electromagnetic fields as  $t \rightarrow -\infty$ . Such an asymptotic behavior

can be studied more precisely in terms of Møller wave operators, as we shall now briefly discuss. We write

$$\mathbf{N} = \begin{pmatrix} 0 & 0 & c\partial_{\mathbf{r}} \times \int_0^\infty d\lambda \sigma(\mathbf{r}, \lambda) \cdots \\ 0 & 0 & 0 & \lambda \\ -c\partial_{\mathbf{r}} \times & 0 & 0 & 0 \\ -\sigma(\mathbf{r}, \lambda) & -\lambda & 0 & 0 \end{pmatrix} = \mathbf{N}_0 + \mathbf{N}_1, \quad (A2)$$

where

$$\mathbf{N}_0 = \begin{pmatrix} 0 & 0 & c\partial_{\mathbf{r}} \times & 0 \\ 0 & 0 & 0 & \lambda \\ -c\partial_{\mathbf{r}} \times & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \end{pmatrix}, \quad (A3)$$

$$\mathbf{N}_1 = \begin{pmatrix} 0 & 0 & 0 & \int_0^\infty d\lambda \sigma(\mathbf{r}, \lambda) \cdots \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\sigma(\mathbf{r}, \lambda) & 0 & 0 & 0 \end{pmatrix}. \quad (A4)$$

Let

$$\mathbf{P}_{\text{aux}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (A5)$$

be the projector upon the auxiliary fields. Then it can be shown by standard methods that the Møller operators

$$\Omega_{\pm} = \lim_{t \rightarrow \pm\infty} \exp(-Nt) \exp(N_0 t) \mathbf{P}_{\text{aux}} \quad (A6)$$

exist in the strong sense (for the underlying Hilbert space and other details, cf. [27]). A quite more elaborate analysis shows that the adjoints  $\Omega_{\pm}^*$  can also be given as strong limits, i.e.,

$$\Omega_{\pm}^* = \lim_{t \rightarrow \pm\infty} \mathbf{P}_{\text{aux}} \exp(-N_0 t) \exp(Nt) \quad (A7)$$

exist, implying that

$$\mathbf{P}_{\text{aux}} \mathbf{F}(t) \xrightarrow{t \rightarrow \pm\infty} \exp(N_0 t) \Omega_{\pm}^* \mathbf{F}(0), \quad (A8)$$

i.e. the motion of the auxiliary fields becomes decoupled from that of the electromagnetic ones for large times. Physically, this can be understood by observing that for a finite dielectric the auxiliary fields do not propagate (they are confined to the dielectric), whereas the electromagnetic fields propagate away. This gives a rigorous underpinning of the existence of the limits in Eq. (3.19). We can arrive at Eq. (3.20), starting from Eq. (A8), noting that

$$\begin{aligned} \Omega_{\pm}^* \mathbf{F}(0) &= \mathbf{P}_{\text{aux}} \mathbf{F}(0) \\ &+ \int_0^{\pm\infty} dt \mathbf{P}_{\text{aux}} \exp(-N_0 t) (\mathbf{N} - \mathbf{N}_0) \exp(Nt) \mathbf{F}(0) \\ &= \mathbf{P}_{\text{aux}} \mathbf{F}(0) + \int_0^{\pm\infty} dt \mathbf{P}_{\text{aux}} \exp(-N_0 t) \mathbf{N}_1 \mathbf{F}(t), \end{aligned} \quad (A9)$$

and working things out, which leads to

$$\begin{pmatrix} \mathbf{F}'_2 \\ \mathbf{F}'_4 \end{pmatrix} = \begin{pmatrix} (\Omega^* \mathbf{F})_2 \\ (\Omega^* \mathbf{F})_4 \end{pmatrix} = \begin{pmatrix} \mathbf{F}_2(0) \\ \mathbf{F}_4(0) \end{pmatrix} + \int_{-\infty}^0 ds \begin{pmatrix} -\sin(\lambda s) \\ \cos(\lambda s) \end{pmatrix} \sigma(\lambda) \mathbf{F}_1(s). \quad (\text{A10})$$

Following the procedure given above, one can show an equivalent expression to Eq. (3.20) valid in the quantum case.

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- [1] E. Snoeks, A. Lagendijk, and A. Polman, Phys. Rev. Lett. **74**, 2460 (1995).
  - [2] Ho Trung Dung, L. Knöll, and D.-G. Welsch, Phys. Rev. A **62**, 053804 (2000).
  - [3] E.M. Lifshitz, Sov. Phys. JETP **2**, 73 (1956).
  - [4] P.W. Milonni, *The Quantum Vacuum* (Academic, Boston, 1994).
  - [5] L.-M. Duan, G. Giedke, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000); J. Lee, M.S. Kim, and H. Jeong, Phys. Rev. A **62**, 032305 (2000); S. Scheel, L. Knöll, T. Opatrný, and D.-G. Welsch, Phys. Rev. A **62**, 43803 (2000).
  - [6] B. Lastdrager, A. Tip, and J. Verhoeven, Phys. Rev. E **61**, 5767 (2000).
  - [7] J.M. Jauch and K.M. Watson, Phys. Rev. **74**, 950 (1948); Phys. Rev. **74**, 1485 (1948).
  - [8] L. Knöll, W. Vogel, and D.-G. Welsch, Phys. Rev. A **36**, 3803 (1987).
  - [9] R.J. Glauber and M. Lewenstein, Phys. Rev. A **43**, 467 (1991); G. Kweon and N. M. Lawandy, Opt. Commun. **118**, 388 (1995).
  - [10] M. Hillery and L.D. Mlodinow, Phys. Rev. A **30**, 1860 (1984); P.D. Drummond, Phys. Rev. A **42**, 6845 (1990).
  - [11] J.J. Hopfield, Phys. Rev. **112**, 1555 (1958).
  - [12] B. Huttner, J.J. Baumberg, and S.M. Barnett, Europhys. Lett. **16**, 177 (1991); B. Huttner and S.M. Barnett, Phys. Rev. A **46**, 4306 (1992).
  - [13] M.S. Yeung and T.K. Gustafson, Phys. Rev. A **54**, 5227 (1996).
  - [14] T. Gruner and D.-G. Welsch, Phys. Rev. A **51**, 3246 (1995);
  - [15] S.M. Dutra and K. Furuya, Phys. Rev. A **57**, 3050 (1998).
  - [16] A. Tip, A. Moroz, and J.-M. Combes, J. Phys. A: Math. Gen. **33**, 6223 (2000).
  - [17] *Photonic Crystals and Light Localization*, Proceedings of the Nato Advanced Study Institute, June 19-30, 2000, Crete, Greece, C. M. Soukoulis, ed. Forthcoming.
  - [18] A. Moroz, Phys. Rev. Lett. **83**, 5274 (1999).
  - [19] R. Matloob and R. Loudon, Phys. Rev. A **52**, 4823 (1995); R. Matloob and R. Loudon, Phys. Rev. A **53**, 4567 (1996); R. Matloob, R. Loudon, M. Artoni, S.M. Barnett, and J. Jeffers, Phys. Rev. A **55**, 1623 (1997).
  - [20] T. Gruner and D.-G. Welsch, Phys. Rev. A **53**, 1818 (1996).
  - [21] Ho Trung Dung, L. Knöll, and D.-G. Welsch, Phys. Rev. A **57**, 3931 (1998).
  - [22] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A **58**, 700 (1998).
  - [23] S. Scheel, L. Knöll, D.-G. Welsch, and S.M. Barnett, Phys. Rev. A **60**, 1590 (1999); S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A **60**, 4094 (1999); *ibid.* **61**, 069901(E) (2000).
  - [24] W.C. Chew, *Waves and Fields in Inhomogeneous Media* (IEEE Press, New York, 1995).
  - [25] A. Moroz, J. Phys.: Condens. Matter **6**, 171 (1994); A. Moroz and C. Sommers, J. Phys.: Condens. Matter **11**, 997 (1999).
  - [26] A. Tip, Phys. Rev. A **56**, 5022 (1997).
  - [27] A. Tip, Phys. Rev. A **57**, 4818 (1998).
  - [28] L. Knöll, S. Scheel, and D.-G. Welsch, *QED in Dispersing and Absorbing Media*, contribution to *Coherence and Statistics of Photons and Atoms*, ed. by J. Peřina, to be published by John Wiley & Sons, Inc.; arXiv: *quant-ph/0006121*.
  - [29] L.D. Landau and E.M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, 1960).